

Weak Scale Inflation and Unstable Domain Walls

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Abstract

Cosmological domain walls produced during phase transition are expected to collapse on a cosmologically safe timescale if Vilenkin's condition is satisfied. We show that the decaying processes of these unstable domain walls should be changed significantly if weak scale inflation takes place. The usual condition for the safe decay of the cosmological domain wall must be changed, depending on their scales and interactions. As a result, the energy scales and explicit breaking terms of such walls must satisfy severe requirements. We also make a brief comment on cosmological structure formation.

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1 Introduction

Many types of discrete symmetries appear in supersymmetric models. They are usually broken at intermediate scales and cause the cosmological domain wall problem, if the walls remain stable. The usual assumption is that non-renormalizable terms induced by gravitational interactions may explicitly break these symmetries and make such walls collapse on a cosmologically safe timescale[1].

On the other hand, many models of supersymmetry breaking involve particles with the masses of order the gravitino mass $m_{3/2}$ and Planck mass suppressed couplings. Coherent production of such particles in the early universe destroys the successful predictions of nucleosynthesis. This problem may be solved by a brief period of weak scale inflation.

In this paper we examine whether the usual criterion for the safe decay of the unstable domain walls can be applied when weak scale inflation takes place. In section 2 and 3 we review the usual condition for unstable domain walls and the basic idea of thermal inflation. In section 4 we show collective examples of discrete symmetries which appear in many supersymmetric models. In section 5 we examine whether the usual criterion for the safe decay of the cosmological domain wall is also applicable for supersymmetric models. We show that the decaying process of these unstable domain walls should be changed significantly if thermal inflation occurs. As a result, the scenario for the safe decay of the cosmological domain walls must be changed, depending on their scales and interactions.

We also make a brief comment on the cosmological structure formation which will be induced by the soft domain walls. If such walls are expanded during weak scale inflation, the conditions for the structure formation will be changed.

2 Collision of the Cosmological Domain Walls

In this section we briefly review how to estimate the value of the pressure (i.e., explicit breaking) to safely remove the walls. The crudest estimate we can make is to insist that the walls are removed before they dominate over the radiation energy density in the universe. When the discrete symmetry is broken by gravitational interactions, the symmetry is

an approximate discrete symmetry. The degeneracy is broken and the energy difference $\epsilon \neq 0$ appears. Regions of higher density vacuum tend to shrink, the corresponding force per unit area of the wall is $\sim \epsilon$. The energy difference ϵ becomes dynamically important when this force becomes comparable to the force of the tension $f \sim \sigma/R$, where σ is the surface energy density of the wall. For walls to disappear, this has to happen before the walls dominate the universe. On the other hand, the domain wall network is not a static system. In general, initial shape of the walls right after the phase transition is determined by the random variation of the scalar VEV. One expects the walls to be very irregular, random surfaces with a typical curvature radius, which is determined by the correlation length of the scalar field. To characterize the system of domain walls, one can use a simulation[2]. The system will be dominated by one large (infinite size) wall network and some finite closed walls (cells) when they form. The isolated closed walls smaller than the horizon will shrink and disappear soon after the phase transition. Since the walls smaller than the horizon size will efficiently disappear so that only walls at the horizon size will remain, their typical curvature scale will be the horizon size, $R \sim t \sim M_p/g_*^{\frac{1}{2}}T^2$. Since the energy density of the wall ρ_w is about

$$\rho_w \sim \frac{\sigma}{R}, \quad (2.1)$$

and the radiation energy density ρ_r is

$$\rho_r \sim g_* T^4, \quad (2.2)$$

one sees that the wall dominates the evolution below a temperature T_w

$$T_w \sim \left(\frac{\sigma}{g_*^{1/2} M_p} \right)^{\frac{1}{2}}. \quad (2.3)$$

To prevent the wall domination, one requires the pressure to have become dominant before this epoch,

$$\epsilon > \frac{\sigma}{R_w} \sim \frac{\sigma^2}{M_p^2}. \quad (2.4)$$

Here R_w denotes the horizon size at the wall domination. A pressure of this magnitude would be produced by higher dimensional operators which explicitly break the discrete symmetry.

The criterion (2.4) seems appropriate, if the scale of the wall is higher than $(10^5 GeV)^3$. For the walls below this scale ($\sigma \leq (10^5 GeV)^3$), there should be further constraints coming from primordial nucleosynthesis. Since the time associated with the collapsing temperature T_w is $t_w \sim M_p^2 / g_*^{\frac{1}{2}} \sigma \sim 10^8 \left(\frac{(10^5 GeV)^3}{\sigma} \right) \text{sec}$, the walls $\sigma \leq (10^5 GeV)^3$ will decay after nucleosynthesis[3]. If the walls are not hidden and can decay into the standard model particles, the entropy produced when walls collide will violate the phenomenological bounds for nucleosynthesis. On the other hand, this simple bound ($\sigma \geq (10^5 GeV)^3$) is not effective for the walls which cannot decay into standard model particles. The walls such as soft domain walls[4, 5], the succeeding story should strongly depend on the details of the hidden components and their interactions. These walls can decay late to contribute to the large scale structure formation.

3 Weak scale inflation

Supersymmetry is probably one of the most attractive extensions of the standard model. In virtue of supersymmetry, the hierarchy can be stabilized against the radiative corrections.

However, overviewing the cosmology of the supersymmetric models, one faces with various difficulties. One of the most obvious and famous problems is the gravitino problem[6]. This problem still exists even if the universe experiences a primordial inflation, since the gravitino is reproduced during the reheating process. The mass of the gravitino depends on the mechanisms of supersymmetry breaking. In the supergravity mediated models of supersymmetry breaking, the gravitino has a mass of the electroweak scale ($m_{3/2} \sim 10^{2-3} \text{GeV}$), and it decays soon after big bang nucleosynthesis. High energy photons produced by the gravitino decay may destroy the usual assumptions for big bang nucleosynthesis. Another example is the gauge mediated supersymmetry breaking models, in which the predicted gravitino mass is much lighter than the supergravity mediated models. The gravitino mass is expected to be $m_{3/2} \sim 10 \text{eV} - 1 \text{GeV}$ and cosmologically stable. If the gravitino mass is larger than 1keV , the universe will be overclosed unless the gravitino is diluted at an earlier epoch.

The gravitino problem is a common feature of the superstring inspired models, because

the moduli fields should play the same role as the gravitino. However, the cosmological moduli problem has a different feature. Because a moduli is a scalar field, it should have the potential which is inevitably flat but raised by the moduli mass $m_\phi \sim m_{3/2}$. In the supergravity mediated models of supersymmetry breaking, the moduli fields decay soon after big bang nucleosynthesis as the gravitino, causing to the same cosmological problem. For the gauge mediated models which predicts lighter mass for the moduli, the energy of the oscillation lasts and overcloses the universe for $m_{3/2} < 100\text{MeV}$, or the decay of the moduli gives too much contribution to the $x(\gamma)$ -ray background spectrum for $m_{3/2} \sim 10^{-1} - 10^4\text{MeV}$.

The most promising way to evade these difficulties is to dilute these unwanted relics after the primordial inflation. One of such dilution mechanisms is the thermal inflation model proposed by Lyth and Stewart[7]. Thermal inflation occurs before the electroweak phase transition and produces entropy before big bang nucleosynthesis, which dilutes the moduli density. During thermal inflation the flaton field (i.e., the inflation field for thermal inflation) is held at the origin by finite temperature effects. The potential energy during thermal inflation is the value V_0 of the flaton potential at the origin, which is of order $m^2 M^2$. With $M \sim 10^{12}\text{GeV}$ and $m \sim 10^2\text{GeV}$, this gives $V_0^{1/4} \sim 10^7\text{GeV}$ which satisfies the condition to avoid the excessive regeneration of light stable fields. Thermal inflation starts when the thermal energy density falls below V_0 which corresponds to a temperature roughly of order $V_0^{1/4}$, and it ends when the finite temperature becomes ineffective at a temperature of order m . The number of e-folds is $N_e^{th} = \frac{1}{2}\ln(M/m) \sim 10$, which is much smaller than the primordial inflation. There is also the intriguing possibility that two or more bouts of thermal inflation can occur in succession, allowing more efficient solution of the moduli problem. In such cases the number of e-folds will be about $N_e = 10 - 25$ [7].

4 Unstable Domain Walls

The discrete symmetry appears in many supersymmetric models. The origins of such symmetries can be traced back to $U(1)_R$ anomaly in the dynamical sector, or the discrete symmetry in the superstring models which appears as the consequence of possible compactification schemes. In this section we make a collective review of such discrete

symmetries. Here we consider a domain wall with the energy scale $\sigma \sim \Lambda_w^3$.

R-changed

In the supergravity mediated models for dynamical supersymmetry breaking, gaugino condensation in the hidden sector of order 10^{10-12}GeV is mediated by gravity, bulk fields or other interactions such as an anomalous $U(1)_X$ field and induces soft supersymmetry breaking terms in the observable sector. In the simplest hidden sector model (supersymmetric Yang-Mills), $U(1)_R$ symmetry in the hidden sector is broken by anomaly and a discrete R symmetry remains. The discrete R symmetry is then spontaneously broken when gaugino condensate. This is a well-known example of the BPS domain wall[8] in the global supersymmetric gauge theory. One may expect that the domain wall structure in the hidden dynamical sector is a common feature of such models for dynamical supersymmetry breaking.

On the other hand, in the gauge mediated models for dynamical supersymmetry breaking, $U(1)_R$ symmetry is strongly connected to the supersymmetry breaking. The presence of an $U(1)_R$ symmetry is a necessary condition for supersymmetry breaking and a spontaneously broken $U(1)_R$ symmetry is a sufficient condition provided two conditions are satisfied. These conditions are genericity and calculability. This means that the domain wall structure in the dynamical sector is not a common feature of the gauge mediated supersymmetry breaking models. However, there are many models which do not satisfy the genericity condition[9] where the $U(1)_R$ symmetry will be anomalous. In these models a discrete R symmetry is implemented and is spontaneously broken at relatively low energy scale of order 10^{5-9}GeV . In these models for dynamical supersymmetry breaking, the dynamical sector may have a domain wall configuration at intermediate scale $\Lambda_w \sim 10^{5-9}\text{GeV}$.

The R-charged domain wall configuration may also appear as the consequence of the spontaneous symmetry breaking of the explicit Z_n^R symmetry, which is sometimes imposed by hand in order to solve phenomenological difficulties such as the μ -problem[3] or the cosmological moduli problem[10]. In these models the scales of the domain walls are determined by phenomenological requirements, typically at $\Lambda_w \sim 10^{5-10}\text{GeV}$.

When the universe undergoes a phase transition associated with spontaneous breaking of such discrete symmetries, domain walls will inevitably form. These domain walls are

generally not favorable if they are stable.

In ref.[11] we have shown that a constant term in the superpotential always breaks the degeneracy of vacua when supergravity is turned on, and that the pressure induced by the constant term satisfies the usual condition for the safe decay of unwanted domain walls. In general, the constant term is required to make the cosmological constant very small, which is an inevitable feature of any phenomenological models for supergravity. The magnitude of the energy difference ϵ induced by the constant term is $\epsilon \sim \sigma^2/M_p^2$ where σ is the surface energy density of the wall. This satisfies the usual condition for the safe decay of the cosmological domain wall. Although the model is technically non-generic because it includes a single term which explicitly breaks the Z_n^R symmetry, it is still a reasonable model for a vanishing cosmological constant. In this sense, the basic idea is similar to the well-known mechanism for the mass generation of the R-axion[12].

Not R-charged

On the other hand, for the walls which do not have R charge, the explicit symmetry breaking term should be added by hand. Since the gravity interaction does not respect global symmetries such as the discrete symmetries we are concerned about, the explicit breaking terms may appear as higher order terms suppressed by the Planck mass. In this case, however, there is no reason to expect that the magnitude of the energy difference appears at their lowest bound $\epsilon \sim \sigma^2/M_p^2$.

5 Unstable Domain Walls and Thermal Inflation

In this section we shall discuss the formation, evolution and collapsing process of the cosmological domain walls paying attention to the changes that should be induced by weak scale inflation.

In general, initial shape of the walls right after the phase transition is determined by the random variation of the scalar VEV. One expects the walls to be very irregular, random surfaces with a typical curvature radius, which is determined by the correlation length of the scalar field. To characterize the system of domain walls, one can use a simulation[2]. The system will be dominated by one large (infinite size) wall network and some finite closed walls (cells) when they are produced. The isolated closed walls smaller

than the horizon will shrink and disappear soon after the phase transition. As a result, only a domain wall stretching across the horizon will remain. The initial distribution of the cosmological domain walls after the primordial inflation is not determined solely from the thermal effect of reheating. In some cases non-linear dynamics of the fields (parametric resonance[13], for example) will be important. Here we do not discuss further on these topics and temporarily make a simple assumption that the walls are produced just after the end of the primordial inflation.

For the walls $\Lambda_w < 10^{11}\text{GeV}$ and $\epsilon \sim \sigma^2/M_p^2$, thermal inflation occurs before they collapse. Such walls may experience a large though not huge number of e-folds. Extended structures arising from such weak scale inflations are not necessarily inflated away. Since the cells of the false vacuum cannot decay soon if they are much larger than the horizon scale², we expect that additional constraints should be required for such walls to decay.

When thermal inflation starts, the initial scale of the domain wall network is the same as the particle horizon. It is of order $H_{th}^{-1} \sim (V_0^{1/2}/M_p)^{-1}$, where H_{th} and V_0 denote the Hubble constant and the vacuum energy during thermal inflation. The cells inflate during thermal inflation, then become the scale of order $l_e = H_{th}^{-1}e^{N_{th}}$. Here N_{th} demotes the number of e-folds of thermal inflation. Since weak scale inflation may occur in succession [7], the number of e-folds N_{th} will be the sum of these succeeding weak scale inflations; $N_{th} \sim 10 - 25$.

At the end of weak scale inflation (at the time $t = t_0$),

$$\left(\frac{l}{d_H}\right)_{t=t_0} \sim e^{N_{th}}, \quad (5.1)$$

where d_H denotes the particle horizon.

After thermal inflation, coherent oscillation of the inflation field ϕ starts. The expansion during this epoch is estimated as[14]

$$l = l_{t=t_0} \left(\frac{\rho_\phi}{V_0}\right)^{-\frac{1}{3}}. \quad (5.2)$$

The horizon size during preheating is estimated as

$$d_H \sim \left(\frac{\rho_\phi^{1/2}}{M_p}\right)^{-1}. \quad (5.3)$$

²Here the bubble nucleation rate is extremely small. We do not consider the false vacuum annihilation induced by the bubble nucleation, because such a scenario is not realistic in our model.

Thus the ratio becomes

$$\left(\frac{l}{d_H}\right)_{t_{rad}>t>t_0} \sim e^{N_{th}} \left(\frac{\rho_\phi}{V_0}\right)^{1/6}. \quad (5.4)$$

Here t_{rad} denotes the time when radiation domination starts. Thereafter l grows like T^{-1} , where T denotes the temperature in the radiation dominated era. Assuming the radiation-dominated expansion, the ratio will be

$$\left(\frac{l}{d_H}\right)_{t>t_{rad}} \sim \left(\frac{l}{d_H}\right)_{t=t_{rad}} \left(\frac{T(t)}{T_D}\right). \quad (5.5)$$

Here T_D denotes the reheating temperature after thermal inflation.

The domain wall network enters the particle horizon at the time t_s when the ratio becomes unity. Assuming that this occurs before radiation energy dominates the universe, ρ_ϕ at the time t_s is

$$\rho_\phi|_{t=t_s} \sim e^{-6N_{th}} V_0. \quad (5.6)$$

In the case that the expansion during thermal inflation is about 10^5 and $V_0^{1/4}$ is about 10^7GeV , $\rho_\phi|_{t=t_s}$ is estimated as $\rho_\phi|_{t=t_s} \sim 10^{-2} (GeV)^4$. Here we assumed that the reheating temperature of the thermal inflation is very low ($T_D < 1 \text{GeV}$) in order to ensure sufficient entropy production. As we have discussed in Sec.2, the walls that do not decay until they dominate the universe must be excluded. In this case, the walls that dominate the universe when they enter the horizon (i.e., $\Lambda_w \geq 10^6 \text{GeV}$) are ruled out. This bound seems very severe, since the walls below this scale should be further constrained by primordial nucleosynthesis. On the other hand, one may think that the walls of the scale $\Lambda_w < 10^7$ should be produced during thermal inflation and suffer less expansion. Including this effect, the above constraint can be relaxed to allow the walls of the scale $\Lambda_w \sim 10^{5-6} \text{GeV}$.

As a result, in many types of supersymmetric theories, walls of the intermediate scale produced just after primordial inflation cannot decay before they dominate the energy density of the universe, even if they satisfy the usual condition. It must also be noted that this result does not depend on the magnitudes of the explicit breaking terms. If the magnitudes of the explicit breaking terms exceed the Vilenkin's lowest bound, the domination by the false vacuum energy begins at earlier epoch. As a result, the situation becomes worse for such larger magnitudes of the explicit breaking terms. Of course, there may be an exception that the explicit breaking terms are quite large so that the walls decay before weak scale inflation. This may happen for the walls $\Lambda_w > V_0^{1/4}$, although the

requirement becomes very severe. For example, when $\Lambda_w = 10^9 \text{GeV}$ the required value of the energy difference is about $\epsilon > 10^5 \sigma^2 / M_p^2$, which is more than 10^5 times larger than the usual bound.

Another way to avoid the domain wall problem is to gauge the discrete symmetry so that there is really one vacuum. However, nontrivial anomaly cancellation conditions must be satisfied. Sometimes it requires fatal constraint on the components of the model.

6 Conclusions and Discussions

In this paper we have studied the decaying processes of unstable domain walls and shown that the processes should be changed significantly if weak scale inflation takes place. As a result, the usual condition for the safe decay of the cosmological domain wall must be changed. For walls which can decay into particles in the standard model, the energy scales of the walls are strongly restricted.

Although the above constraints looks severe, there are other possibilities related to the structure formation; cosmology with ultra-light pseudo-Nambu-Goldstone bosons[4, 5]. Contrary to the ordinary types of cosmological domain walls which we have discussed above, expansion during thermal inflation is a good news for such scenarios. Late decay of the soft domain walls can be realized as a natural consequence of thermal inflation, and can contribute to the large scale structure formation. We will study this topic in the next paper.

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